References

[1] Seide, P., "Axisymmetrical buckling of circular cones under axial compression," J. Appl. Mech. **23**, 625–628 (1956).

[2] Seide, P., "The stability of thin conical frustrums subjected to axial compression and internal or external uniform hydrostatic pressure," Space Technology Labs. Inc., Rept. AF BMD-TR-61-37 (March 1961).

[3] Singer, J., "Buckling of conical shells under axisymmetrical external pressure," J. Mech. Eng. Sci., **3**, 330–339 (1961).

[4] Singer, J., "The effect of axial constraint on the instability of thin circular cylindrical shells under uniform axial compression," Intern. J. Mech. Sci. **4**, 253–258 (1962).

[5] Lackman, L. and Penzien, J., "Buckling of circular cones under axial compression," J. Appl. Mech. **27**, 458–460 (1960).

[6] Weingarten, V. I., Morgan, E. J., and Seide, P., "Final report on the development of design criteria for elastic stability of thin shell structures," Space Technology Labs., Los Angeles, TR-60-0000-19425 (December 1960).

[7] Schnell, W. and Schiffner, K., "Experimentelle Untersuchungen des Stabilitätsverhaltens von dünnwandigen Kegelschalen unter Axiallast und Innendruck," Deut. Vers. Luft Raumfahrt, DVL Rept. 243 (November 1962).

Supersonic and Hypersonic Flow of an Ideal Gas around an Elliptic Nose

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THE direct method of Dorodnitsyn[1] and Belotserkovskii[2–4] has been applied to obtain a first-order (one-strip) solution to the inviscid equations of the mixed subsonic-supersonic flow of a perfect gas about the nose of an elliptic cylinder. The equations were written[5] in coordinates along and normal to the body surface, as first used by Traugott[6] (see Fig. 1). This system enables bodies of arbitrary convex shape to be considered. In addition, by writing the equations in terms of the inverse of Mach number, an explicit solution was obtained for infinite Mach number. Thus the dimensionless freestream velocity becomes, using Traugott's notation,[6]

$$v_\infty^2 = \frac{1}{1 + 1/[(\gamma - 1)/2]M^2_\infty}$$

and the components of the velocity at the shock wave are

$$v_{s\delta} = v_\infty[\cos\theta - (1 - \epsilon)\cos\chi\cos(\theta + \chi)]$$

$$v_{n\delta} = v_\infty[-\sin\theta + (1 - \epsilon)\cos\chi\sin(\theta + \chi)]$$

Table 1 Stagnation-point shock standoff distance, first-order, one-strip Dorodnitsyn-Belotserkovskii (D-B) theory

	M_∞			
a/b	3.0	6.8	30	∞
0.5	0.4711	0.3166	...	0.2790
1.0	0.7016	0.4238	...	0.3728
2.0	0.8489	0.4946	...	0.4319
5.0	0.9306	0.5282	0.4599	0.4564
10.0	0.9446	0.5335	...	0.4604

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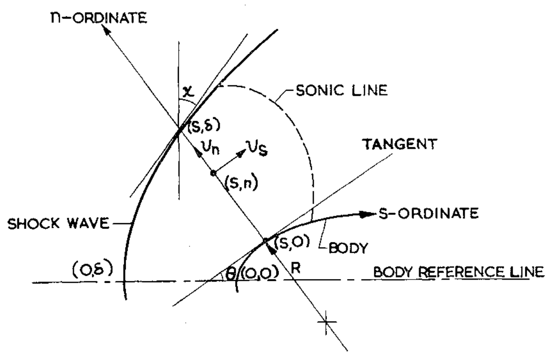

Fig. 1 δ-n **coordinate system, Traugott.[6]**

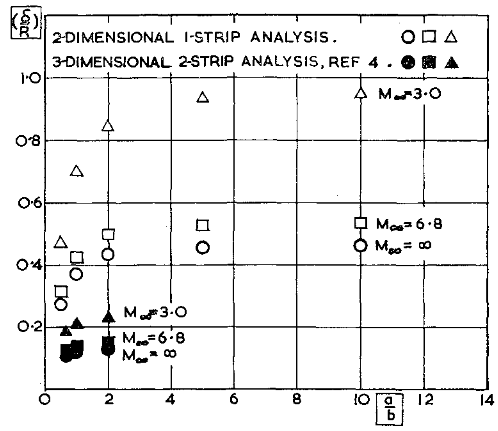

Fig. 2 Stagnation-point shock standoff distance, Dorodnitsyn-Belotserkovskii (D-B) method.

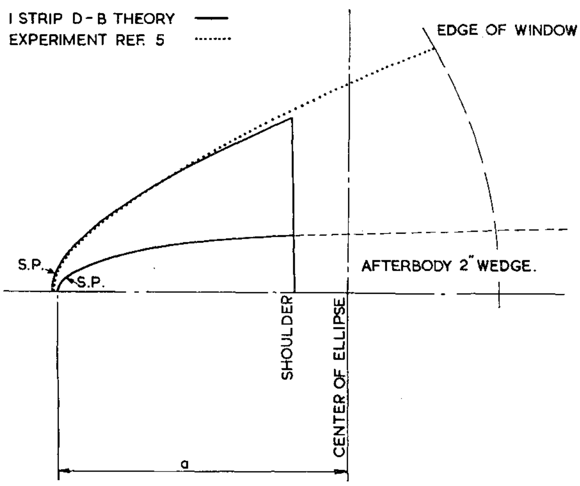

Fig. 3 Comparison of theoretical and experimental shock shape on elliptic cylinder, $a/b = 5$, $a = 3.046$ in., $R_0 = 0.122$ in., $M_\infty = 6.8$, $RN_\infty = 6.72 \times 10^4$/in.

where

$$\epsilon = [(\gamma - 1)/(\gamma + 1)](1 + w) = \rho_\infty/\rho_\delta$$

$$w = \frac{1}{[(\gamma - 1)/2]M_\infty^2\cos^2\chi}$$

The numerical analysis was performed on the Control Data Corporation 1604 at the University of Minnesota. Extrapolation to pass through the sonic singularity was made from 95 to 105% of sonic velocity and used a second-degree curve fit. Numerical integration was performed using the Adams-Moulton routine[7] with fixed step, the size of which was found to give

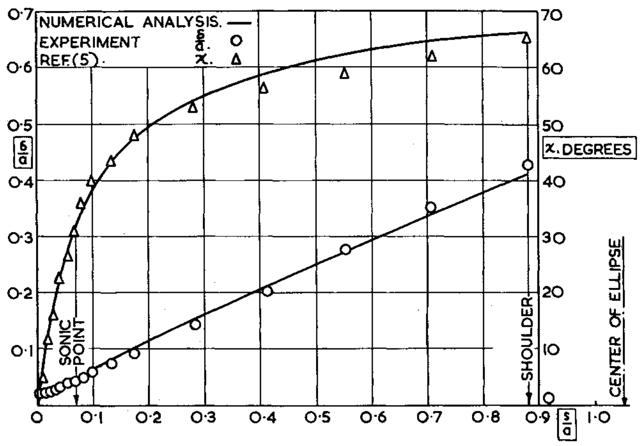

Fig. 4 Comparison of theoretical and experimental shock shape parameters δ and χ on elliptic nose, $a/b = 5$, $M_\infty = 6.8$, $RN_\infty = 6.72 \times 10^4$/in.

convergence of the solution if the length from stagnation point to sonic point was divided into 200 or more steps.

The axis ratio of the ellipse a/b was varied from 0.5 to 10, and Mach numbers were 3, 6.8, and ∞. The results for shock standoff distance (Table 1) are rounded off at four significant figures, but in many cases were carried to eight places. Figure 2 shows that at hypersonic speeds the stagnation-point shock standoff distance depends only upon the nose radius of curvature for axis ratios above 4 or 5. The limiting value appears to be about 0.46 of the nose radius of curvature. A cross-plot of Belotserkovskii's results for axisymmetric ellipsoids[4] is included for comparison.

Experimentation[5] at $M_\infty = 6.8$ and Reynolds number $RN_\infty = 6.7 \times 10^4$/in. on an elliptic cylinder having $a/b = 5$ shows agreement with the preceding solution for shock standoff distance in the region between the stagnation point and sonic point within the limit of error of measurement. This was 7% on δ/R at the stagnation point, falling to 4% at the sonic point. Figures 3 and 4 show the experimental shock shape compared with one-strip theory for the elliptic cylinder.

References

[1] Dorodnitsyn, A. A., "A contribution to the solution of inviscid problems of transonic aerodynamics," *Advances in Aeronautical Sciences* (Pergamon Press, London, 1959), Vol. 2, pp. 832–843.

[2] Belotserkovskii, O. M., "Flow with a detached shock wave about a symmetrical profile," Prikl. Mat. Mekh. **22**, 206–216 (1958).

[3] Belotserkovskii, O. M., "Flow past a circular cylinder with a detached shock wave," Vychislitel'nala Mat. **3**, 149–185 (1958); trans. as Avco Tech. Memo. RAD-9-TM-59-66 (September 30, 1959).

[4] Belotserkovskii, O. M., "On the calculation of flow past axisymmetric bodies with detached shock waves using an electronic computing machine," Prikl. Mat. Mekh. **24**, 511–517 (1960).

[5] Archer, R. D., "Investigation of heat transfer and viscous flow phenomena on delta wings at Mach 7," Ph.D. Thesis, Univ. of Minnesota; Thesis Advisor, R. Hermann (December 1963).

[6] Traugott, S. C., "An approximate solution of the direct supersonic blunt-body problem for arbitrary axisymmetric shapes," J. Aerospace Sci. **27**, 361–370 (1960).

[7] Dolid, L., "SDAM, Adams-Moulton, Runge-Kutta differential equations subroutine," Minneapolis-Honeywell Memo. MR 8029 (April 11, 1962).

Technical Comments

Comment on "Mass and Magnetic Dipole Shielding against Electrons of the Artificial Radiation Belt"

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THE article by Bhattacharjie and Michael[1] shows that magnetic radiation shielding against electrons of the artificial belt is attractive when compared to solid shielding. It appears that they have chosen a relatively unfavorable configuration for their magnetic shield; as a result of this, they have emphasized a relatively unimportant aspect of the design, and substantially understated the attractiveness of their concept.

The shielded volume V around a dipole is proportional to $M^{3/2}$, where M is the magnetic moment of the dipole. M, in turn, is proportional to Ia^2 where I is the total circulating current and a the radius of the coil. The mass of structural material m_{st} required to contain the magnetic stresses is[2–4] proportional to the magnetic energy stored by the device; this varies as I^2a for fixed geometrical ratios. For a given maximum current density of the superconducting material, the mass of superconductor m_{sc} required is proportional to Ia. Thus, for a given shielded volume (or magnetic moment), both masses can be indefinitely reduced by reducing I and increasing a. Bhattacharjie and Michael have worked, however, at a fixed magnetic field strength of 50 kgauss. That is, they have fixed the ratio I/a. From the foregoing, it can be seen that this implies that $m_{st} \propto V^{2/3}$, and this proportionality appears in their Eq. (6) and also in their Fig. 2. Much lower weights than they quote can be achieved by reducing the level of both the current and magnetic field, and by enlarging the geometry. The optimization actually performed by Bhattacharjie and Michael deals only with the aspect ratio of the solenoid providing the magnetic moment. Based on considerations of structural efficiency, they arrive at the conclusion that the optimum aspect ratio is roughly square. We shall see below that structural considerations are, in any event, unimportant for the shielding purposes under consideration.

One cannot, of course, achieve indefinitely lower values for m_{st} and m_{sc} by reducing I and increasing a. The shielded volume declines sharply if a is larger than the Stormer radius c_{st} because the magnetic field in the shielded volume becomes significantly different from the field due to a dipole. Minimum weight for a given shielded volume occurs when these lengths are roughly equal, thus

$$a/c_{st} = (4\pi\ pa^2/eM)^{1/2} = (4p/\mu_0 eI)^{1/2} \approx 1$$

in mks units. For 10 Mev electrons, the momentum to charge ratio p/e is 0.035. Therefore, a shield of any size designed to protect against 10 Mev electrons should operate with fixed total current of about 10^5 amp.

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